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Q1.

a) $\frac{\partial \rho}{\partial t} = 0 \Rightarrow \int J\, ds = I$ $\int J\, ds = \frac{d}{dt}\int \rho\, dv \Rightarrow \int J\, ds = \int \frac{d\rho}{dt}\, dv$

$\int J\, ds = \frac{dQ}{dt}$

$J = \frac{\sigma D}{\varepsilon}$

$\Rightarrow \nabla \cdot J - \frac{\partial \rho}{\partial t} = 0 \Rightarrow \frac{\sigma}{\varepsilon}\nabla D - \frac{\partial \rho}{\partial t} = 0 \Rightarrow \frac{\sigma}{\varepsilon}\rho \cdot \cancel{\frac{\partial \rho}{\partial t}} = 0 \Rightarrow \boxed{\rho = 0}$

b) $\nabla^2 V = \rho \Rightarrow \nabla^2 V = 0$ (laplace)

$\frac{1}{r}\cdot\frac{\partial}{\partial r}\left(r\frac{\partial V}{\partial r}\right) = 0 \Rightarrow V(r) = A\ln r + B$

$r = a \Rightarrow V(a) = V_0 = A\ln a + B$

$r = b \Rightarrow V(b) = 0 = A\ln b + B$

$\therefore B = -A\ln b = \frac{-V_0\ln(b)}{\ln\left(\frac{a}{b}\right)}$

$A = \frac{V_0}{\ln\left(\frac{a}{b}\right)}$

$V(r) = \frac{V_0\ln r}{\ln\left(\frac{a}{b}\right)} - \frac{V_0\ln(b)}{\ln\left(\frac{a}{b}\right)} = \frac{V_0(\ln(r) - \ln(b))}{\ln\left(\frac{a}{b}\right)}$

$\boxed{V(r) = \frac{V_0\ln\left(\frac{r}{b}\right)}{\ln\left(\frac{a}{b}\right)}} \Rightarrow E = -\nabla V = \frac{-V_0 b}{r}\cdot\frac{1}{b}\cdot\frac{1}{\ln\left(\frac{a}{b}\right)} \Rightarrow \boxed{E(r) = \frac{-V_0}{r\ln\left(\frac{a}{b}\right)}}$

c) $\int E\, ds = \frac{Q}{\varepsilon_0} \Rightarrow E(a)\Big|_0^l \Rightarrow \rho_s =$

$Q = \varepsilon_0\int_0^l \frac{-V_0}{a\ln\left(\frac{a}{b}\right)} \underbrace{2\pi a\, dl}_{\text{area superfície}} = \frac{-2\pi V_0 l\varepsilon}{\ln\left(\frac{a}{b}\right)} \Rightarrow \frac{C}{l} = \frac{Q}{lV(r=a)} = \frac{-2\pi\varepsilon}{\ln\left(\frac{a}{b}\right)}$

$\sim \frac{Q}{lV_0}$

d) $J = \sigma E = \dfrac{-V_0\sigma}{r\ln\left(\frac{a}{b}\right)}$

$$I = \int J\,ds = \int_a^b \frac{-V_0\sigma}{r\ln\left(\frac{a}{b}\right)}^{?} \cdot 2\pi r\,dr = \frac{-V_0\sigma}{\ln\left(\frac{a}{b}\right)}\cdot(b-a) = \boxed{\frac{V_0\sigma(a-b)}{\ln\left(\frac{a}{b}\right)}}$$

$$R = \frac{V_0}{I} = \boxed{\frac{\ln\left(\frac{a}{b}\right)}{\sigma(a-b)}}$$

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Q2. $J = \hat{z}\,\dfrac{J_0}{r}\,\mathrm{sen}\left(\dfrac{\pi r}{a}\right)$

a)

$$\int J\,ds = I \Rightarrow \int_0^a \frac{J_0}{\cancel{r}}\,\mathrm{sen}\left(\frac{\pi r}{a}\right) 2\pi \cancel{r}\,dr = I \Rightarrow J_0\left(\frac{a}{\pi}\right)\cdot\left(-\cos\left(\frac{\pi r}{a}\right)\right)\Big|_0^a = \frac{I}{2\pi}$$

$$-J_0\Big[\overbrace{\cos\pi - \cos 0}^{-2}\Big] = \frac{I}{2\pi}\cdot\frac{\pi}{a} \Rightarrow \boxed{J_0 = \frac{I}{4a}}$$

b)

$$\int H\,dl = I_{enc} \Rightarrow \overset{r>a}{B} = \frac{\mu I}{2\pi r} = \boxed{\frac{\mu}{2\pi r}\int \frac{I}{4ar}\cdot\mathrm{sen}\left(\frac{\pi r}{a}\right)ds} \ldots$$

c)

$$\int B\,dl = \mu_0 I_{enc} \Rightarrow \boxed{B = \frac{\mu_0 I\left[1-\cos\left(\frac{\pi r}{a}\right)\right]}{4\pi r}}$$

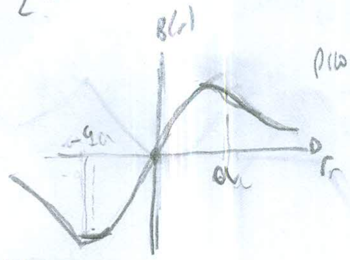
$$I_{enc} = \int_0^r \cancel{J}\,ds = \frac{I}{4a}\int_0^r \frac{1}{\cancel{r}}\,\mathrm{sen}\left(\frac{\pi r}{a}\right)\cdot 2\pi \cancel{r}\,dr = \frac{I}{\cancel{4a}_2}\cdot\left(\frac{\cancel{a}}{\cancel{\pi}}\right)\left[-\cos\left(\frac{\pi r}{a}\right) + 1\right]\cdot\cancel{2\pi}$$

$$= \boxed{\frac{I}{2}\left[1 - \cos\left(\frac{\pi r}{a}\right)\right]}$$

$\mu_0 I\cdot\left(\frac{\pi}{a}\right)\cdot\mathrm{sen}\left(\frac{\pi r}{a}\right) = 0$

d) $B(0) = 0$

$B(a) = \dfrac{\mu_0 I}{2\pi a}$



$(4\pi r)\cdot\left(\mu_0 I\cdot\frac{\pi}{a}\cdot\sin\left(\frac{\pi r}{a}\right)\right) - \left[\mu_0 I\left(1-\cos\left(\frac{\pi r}{a}\right)\right)\cdot 4\pi\right] = 0$

$\left(\frac{r\pi}{a}\sin\left(\frac{\pi r}{a}\right)\right) - \left(4 - 4\cos\left(\frac{\pi r}{a}\right)\right) = 0$

$\boxed{2}$

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Q3

a) $p = \frac{h}{\lambda}$ $n\lambda = 2\pi r$ $\frac{nh}{p} = 2\pi r \Rightarrow \boxed{n\hbar = m_e v \cdot r}$ ✓

$v = \frac{n\hbar}{m_e r} \Rightarrow$

b) $\frac{e^2}{4\pi\varepsilon_0 r^2} = \frac{m}{r}\left(\frac{n\hbar}{m_e r}\right)^2 \Rightarrow \boxed{r_n = \frac{4\pi\varepsilon_0 n^2\hbar^2}{e^2 m_e}}$ ✓

$$E = \frac{1}{2}m\left(\frac{n\hbar}{m_e r}\right)^2 - \frac{e^2}{4\pi\varepsilon_0 r} = \frac{1}{2}\frac{n^2\hbar^2 \cdot e^4 m_e^2}{m_e \cdot (4\pi\varepsilon_0 n^2\hbar^2)^2} - \frac{e^2}{(4\pi\varepsilon_0)^2 n^2\hbar^2} e^2 m_e$$

$$E = \frac{1}{2}\frac{e^4 m_e}{(4\pi\varepsilon_0)^2 n^2\hbar^2} - \frac{e^4 m_e}{(4\pi\varepsilon_0)^2 n^2\hbar^2} = \boxed{-\frac{e^4 m_e}{8(\pi\varepsilon_0)^2 n^2\hbar^2}}$$ ✓

c) $E_1 = +13,6$ $\frac{13,6}{4,0}$ $1,5$ $13,6$

$$\boxed{E_3 = \frac{E_1}{n^2} = \frac{-13,6}{9} = -1,5\ eV}$$

d) $E_2 = -3,6\ eV$

$\Rightarrow \Delta E = 2,75\ eV$

$E_4 = \frac{-13,6}{16} = -0,85$

$$E = \frac{hc}{\lambda} \Rightarrow \lambda = \frac{hc}{2,75} = \frac{4,14 \cdot 10^{-15} \cdot 3 \cdot 10^8}{2,75}$$

$\lambda = 4,14 \cdot 10^{-7} \Rightarrow \boxed{414\ nm}$

ultravioleta visível

Q4 work required to remove electron

a) $K_{max} = h\nu - W$ $\qquad \underline{K_{max} = eV_0}$

$K_{max} = h\nu - w_0$

work function, min energy needed to remove

$V_0 = \frac{h\nu}{e} - \frac{w_0}{e}$ slope fig 2 is h (eV)

60 ⟨ 16
120 0,37

$$h = \frac{(1-0,4)}{(7-5,4)10^{14}} = \frac{0,6 \cdot 10^{-14}}{1,6} = \underline{3,7 \cdot 10^{-15} \text{ eV}}$$

b) $w_0 = h \cdot 4,5 \cdot 10^{14}$ $\quad (V_0 = 0)$

$$\boxed{w_0 = 4 \cdot 4,5 \cdot 10^{-1} = 1,8 \text{ eV}}$$

c) $K_{max} = 4 \cdot 10^{-15} \cdot 10^{15} - 1,8 = \underline{2,2 \text{ eV}}$

d) classicamente 0 ?

não prevê o aumento da freq. mas da intensidade recebe a energia e tb tem o limite mín da freq.

Q5.

a) $\boxed{T_f = \frac{T_2 + T_1}{2}} \rightarrow -Q_A = -mc(T_f - T_A) = mc(T_f - T_B) = Q_B$ $\quad \boxed{2T_f = T_A + T_B}$

$\Delta T_f = ($

$\Delta S = \Delta S_A + \Delta S_B$
$= C_p \left[\ln\left(\frac{T_f^2}{T_1 T_2}\right) \right]$

$\Delta S = \int_{T_i}^{T_f} \frac{mc \, dT}{T}$

b) $\frac{C_p}{T} = \frac{\partial S}{\partial T} = \Delta S = C_p \ln\left(\frac{T_f}{T_i}\right)$ $\quad$ or $\quad dQ = \frac{mc\,dT}{T} \Rightarrow dS = \frac{dQ}{T}$

$\boxed{4}$ $\Delta S_A = C_p \ln\left(\frac{T_f}{T_A}\right)$ $\quad \Delta S_B = C_p \ln\left(\frac{T_f}{T_B}\right)$

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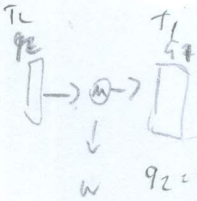
Procesos reversivos

c) estado inicial = estado final $\Delta U = 0$ $q_1 = w + q_2$

$s_A = \frac{Q_A}{T_1} = \frac{Q_B}{T_2} = s_B \Rightarrow \Delta S \geq 0$

T_2, q_2, $\downarrow q_1$, $A \to w$, $\downarrow q_2$



w $q_2 = w + q_1$

$$\int_{T_2}^{T_f} \frac{Q_2}{T_2} - \int_{T_1}^{T_f} \frac{Q_1}{T_1} \geq 0 \Rightarrow \int_{T_2}^{T_f} \frac{c_p\,dT}{T_2} - \int_{T_1}^{T_f} \frac{c_p\,dT}{T_1} \geq 0 \Rightarrow \ln\left(\frac{T_f^2}{T_1 T_2}\right) \geq 0$$

$$\frac{T_f^2}{T_1 T_2} \geq 1 \Rightarrow \boxed{T_f \geq \sqrt{T_1 T_2}}$$

d) $w = q_2 - q_1$

$$= c_p \Delta T_A - c_p \Delta T_B = c_p \left[\overbrace{(T_1 - T_f) - (T_f - T_2)}^{\text{maior} - \text{menor}} \right] \Rightarrow$$

$$\boxed{w = c_p (T_2 + T_1 - 2T_f)}$$

Q6

a) $L = \frac{1}{2}\mu \dot{r}^2 + \frac{1}{2}\mu r^2 \dot{\theta}^2 - U(r) \to \boxed{\frac{d^2u}{d\theta^2} + u = -\frac{\mu}{l^2 u^2} F\left(\frac{1}{u}\right)}$

$\frac{\alpha}{r} = 1 + \varepsilon \cos\theta$

$\varepsilon > 1$ hyperbola $\Rightarrow E > 0$

$0 < \varepsilon < 1$ (ellipse planetary motion)

$L = r_\oplus \times m v$

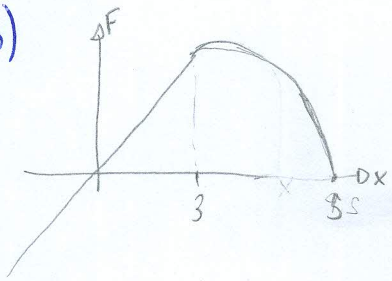
$= r_\oplus \cdot m \cdot v_0 \sin 30^\circ$

Q7. a) $F = -\cancel{K}x \Rightarrow U = \frac{m\omega^2 x^2}{2} \Rightarrow 1mJ = \frac{450g \cdot \omega^2 \cdot 1cm^2}{2}$

$\omega^2 = \frac{K}{m}$

$10^{-3} = \frac{0,450 \cdot \omega^2 \cdot 10^{-4}}{2} \Rightarrow \omega = \sqrt{\frac{20}{0,450}}$ $\boxed{K = \omega^2 m}$

b)



c) $\frac{Kx^2}{2} = \frac{m v_{max}^2}{2} \Rightarrow \boxed{v_{max}^2 = \frac{K(3 \cdot 10^{-2})^2}{m}}$

d) $\frac{Kx^2}{2} + \frac{mv^2}{2} = \frac{m v_{max}^2}{2} \Rightarrow \boxed{\frac{K(2 \cdot 10^{-2})^2}{2} + \frac{mv^2}{2} = \frac{m v_{max}^2}{2}}$

e) $\frac{\cancel{K}x^2}{\cancel{2}} = \frac{K(5 \cdot 10^{-2})^2}{\cancel{2}}$ $x = -5cm$?

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Q8.

a) $$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2\Psi(x)}{\partial x^2} = E\Psi(x)}$$ $0<x<a$

b) $$\frac{\partial^2\Psi(x)}{\partial x^2} + k^2\Psi(x) = 0 \Rightarrow k^2 = \frac{2mE}{\hbar^2} \Rightarrow \boxed{E_n = \frac{n^2\pi^2\hbar^2}{a^2 2m}}$$

$$\Psi(x) = A\cos(kx) + B\sin(kx)$$

$$x=0=a \Rightarrow \Psi=0 \Rightarrow \Psi(x) = B\sin\left(\frac{n\pi}{a}x\right) \Rightarrow \boxed{\Psi_n = \sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)}$$

$$k = \frac{n\pi}{a}$$

$$\int_0^a |\Psi(x)|^2 = 1 \Rightarrow \boxed{B = \sqrt{\frac{2}{a}}}$$

c) $$\frac{\partial^2\Psi(x)}{\partial x^2} + \Psi(x)\left(W_0\sin\left(\frac{\pi x}{a}\right) - k^2\right) = 0$$

$|x|<<1$

$(1+x)^k = 1+kx$

estado fundamental

$$E_n^{(1)} = \int_0^a W_0\sin\left(\frac{\pi x}{a}\right)\cdot\frac{2}{a}\sin^2\left(\frac{n\pi x}{a}\right)dx$$

$$\boxed{E_n^{(1)} = \langle\Psi_n^0|H'|\Psi_n^0\rangle}$$

$$E_n^{(1)} = \frac{2}{a}\left[\int_0^a \frac{W_0\sin\left(\frac{\pi x}{a}\right)}{2} - \frac{\overbrace{W_0\sin\left(\frac{\pi x}{a}\right)\cos\left(\frac{2\pi x}{a}\right)}^{\frac{1}{2}\sin\left(\frac{3\pi x}{a}\right)-\sin\left(\frac{\pi x}{a}\right)}}{2}\right]dx$$

$$= \frac{2}{a}\left[\frac{W_0}{2}\cdot\left(\frac{-2a}{\pi}\right) - \right.$$

$$\boxed{E_n = E_n^{(0)} + E_n^{(1)}}$$

Q9. a) $\hat{n}\cdot\sigma = \sigma_x \sin\theta\cos\phi + \sigma_y \sin\theta\sin\phi + \sigma_z\cos\theta$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\sin\theta\cos\phi + \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\sin\theta\sin\phi + \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\cos\theta$$

$$\boxed{S_n = \frac{\hbar}{2}\begin{pmatrix} \cos\theta & e^{-i\phi}\sin\theta \\ \sin\theta\, e^{i\phi} & -\cos\theta \end{pmatrix}}$$

b) $(\cos\theta - \lambda)(-\cos\theta - \lambda) - \sin^2\theta = 0$

$\lambda^2 - 1 = 0 \Rightarrow \boxed{\lambda = \pm 1}\ \frac{\hbar}{2}$

c) $$\frac{\hbar}{2}\begin{bmatrix} \cos\theta + 1 & e^{-i\phi}\sin\theta \\ & \end{bmatrix}\begin{bmatrix} a \\ b \end{bmatrix} = \frac{\hbar}{2}\begin{bmatrix} a \\ b \end{bmatrix}$$

$\sin^2\left(\frac{\theta}{2}\right) = \frac{1}{2} - \frac{1}{2}\cos(\theta)$

$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\cos\theta}{2}}$

$$\underbrace{(\cos\theta - 1)}^{2\sin^2\left(\frac{\theta}{2}\right)} a + e^{-i\phi}\sin\theta\, b = \frac{\hbar}{2} a$$

$$\left(\cancel{\frac{\hbar}{2}} + 1 - \cos\theta\right) a = e^{-i\phi}\sin\theta\, b \Rightarrow \boxed{\begin{bmatrix} \cos\left(\frac{\theta}{2}\right) \\ e^{i\phi}\sin\left(\frac{\theta}{2}\right) \end{bmatrix}}$$

$$\left(\cancel{\frac{\hbar}{2}} + 2\sin^2\left(\frac{\theta}{2}\right)\right) a = e^{-i\theta}\sin\theta\, b$$

$$\frac{\hbar}{2}\cancel{\cos\left(\frac{\theta}{2}\right)} + 2\sin^2\left(\frac{\theta}{2}\right)\cdot\cos\left(\frac{\theta}{2}\right) = e^{-i\phi}\sin\theta\, b$$

$$\frac{\hbar}{2}\cancel{\cos\left(\frac{\theta}{2}\right)} + \cancel{2}\cdot\sin\left(\frac{\theta}{2}\right)\cdot\frac{1}{2}\cdot\sin\theta = e^{-i\phi}\sin(\theta)\, b$$

$$\boxed{b = e^{i\phi}\sin\left(\frac{\theta}{2}\right)}$$

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d) $\theta = 60°$ $\phi = 45°$ $\chi_+ = \begin{bmatrix} \frac{\sqrt{3}}{2} \\ \left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right)\cdot\frac{1}{2} \end{bmatrix} = \frac{1}{2}\begin{bmatrix} \sqrt{3} \\ \sqrt{2}(1+i) \end{bmatrix}$

$e^{i\frac{\pi}{4}} = \cos\left(\frac{\pi}{4}\right) + i\sin\left(\frac{\pi}{4}\right)$

$= \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

$\langle S_n^+ | S_z^+ \rangle = \frac{1}{2}\begin{bmatrix} \sqrt{3} & \sqrt{2}(1 \mp i) \end{bmatrix} \cdot \frac{1}{2}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \frac{\sqrt{3}}{2}$

$\boxed{P(S_n^+) = |\langle S_n^+ | S_z^+ \rangle|^2 = \frac{3}{4}}$

?

$\left(\frac{k_B T}{c}\right)$ $\int_{-\infty}^{\infty} e^{-\alpha t^2} = \left(\frac{\pi}{\alpha}\right)^{\frac{1}{2}}$

$E = cp$

Q12

a) $\varepsilon = cp$. $Z = \iiint_V \iiint_{-\infty}^{\infty} e^{-\beta c(p_x + p_y + p_z)} \, dp_x \, dp_y \, dp_z \, dV$

$Z = \frac{1}{N!}\frac{V^N}{h^{3N}} \int d^3p \, e^{-\beta |p| c} = \frac{1}{N!}\frac{V^N}{h^{3N}} \cdot \frac{4\pi}{(\beta c)^2}\int_0^{\infty} dp \, e^{-\beta c p} = \boxed{\frac{1}{N!}\frac{(8\pi V)^N}{\left(\frac{hc}{k_B T}\right)^{3N}}}$

b) $P = -\left(\frac{\partial F}{\partial V}\right)_T \Rightarrow F = U - TS = \boxed{-k_B T \ln Z}$

c) $F = U + TS \Rightarrow S = \frac{U - F}{T} \Rightarrow U = k_B \frac{\partial}{\partial \beta} \ln Z$ ou $S = -\left(\frac{\partial F}{\partial T}\right)_V$

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